Last time: 2nd Fundamental Thm of Calculus

2FTC:

If f is a continuous fcn on an interval containing a, $x \in \mathbb{R}$, then If we define

$$F(x) = \int_a^x f(t)\,dt,$$

Then F is differentiable and $F'(x) = f(x)$.

Example:

① Let $g(x) = \int_0^x \sin(\theta^2)\,d\theta$.

Compute $g'(x)$:

$$g'(x) = \sin(x^2)$$

② Let $h(x) = \int_0^{e^x} \sin(\theta^2)\,d\theta$

Compute $h'(x)$.

$$h'(x) = \sin\left((e^x)^2\right) \cdot (e^x)'$$

$$= \boxed{\sin(e^{2x}) \cdot e^{x}}.$$

③ Let $B(x) = \int_x^1 t\,2^t\,dt$

Find $B'(x)$.

$$B(x) = -\int_1^x t\,2^t\,dt$$

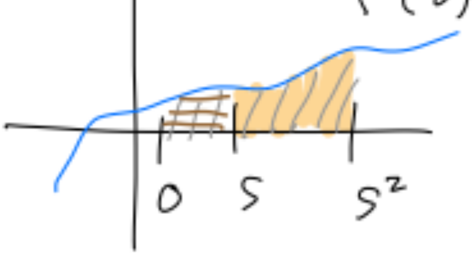
$$B'(x) = -(x2^x)$$

$$= \boxed{-x2^x}.$$

④ Find the derivative of

$$F(s) = \int_s^{s^2} e^{t^2}\,dt$$

$$F(s) = \int_0^{s^2} e^{t^2}\,dt - \int_0^{s} e^{t^2}\,dt$$



$$\Rightarrow F'(s) = e^{(s^2)^2} \cdot (2s) - e^{s^2}$$

$$= \boxed{2se^{s^4} - e^{s^2}}$$

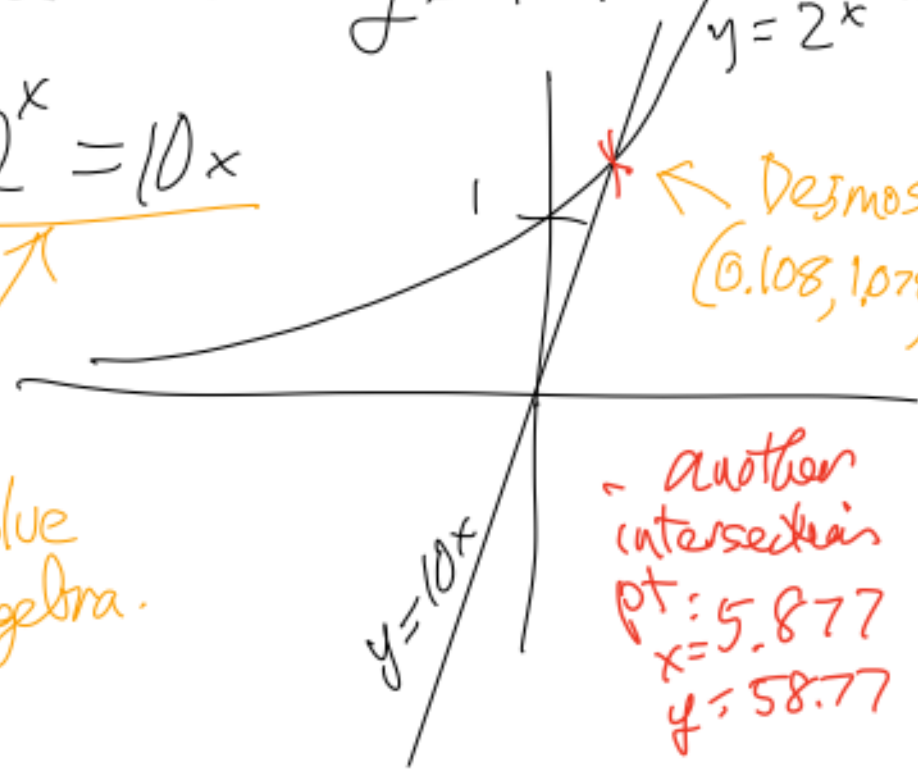
Application of Calculus:

Newton's Method of Finding roots (solving equations).

<u>Example</u>: Where do the graphs of $y = 2^x$ and $y = 10x$ intersect?

$$2^x = 10x$$

Can't solve using algebra.



Desmos: $(0.108, 1.078)$

another intersection pt: $x = 5.877$, $y = 58.77$

Question: How did Desmos calculate the solution to the equation?

To solve, let

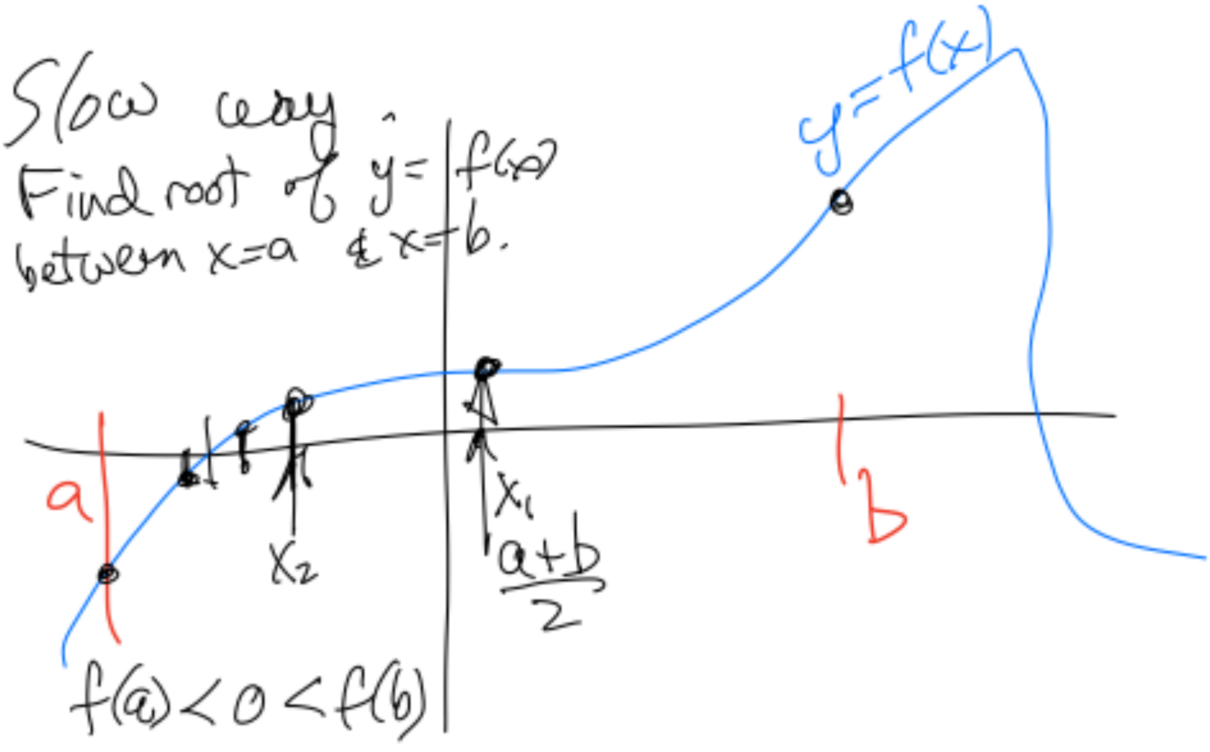
$$f(x) = 2^x - 10x \Leftarrow \text{look}$$

where it is zero.

How to find roots fast.
(where $f(x) = 0$)

- Slow way:

Find root of $y = f(x)$
between $x = a$ & $x = b$.



Bisection method: If

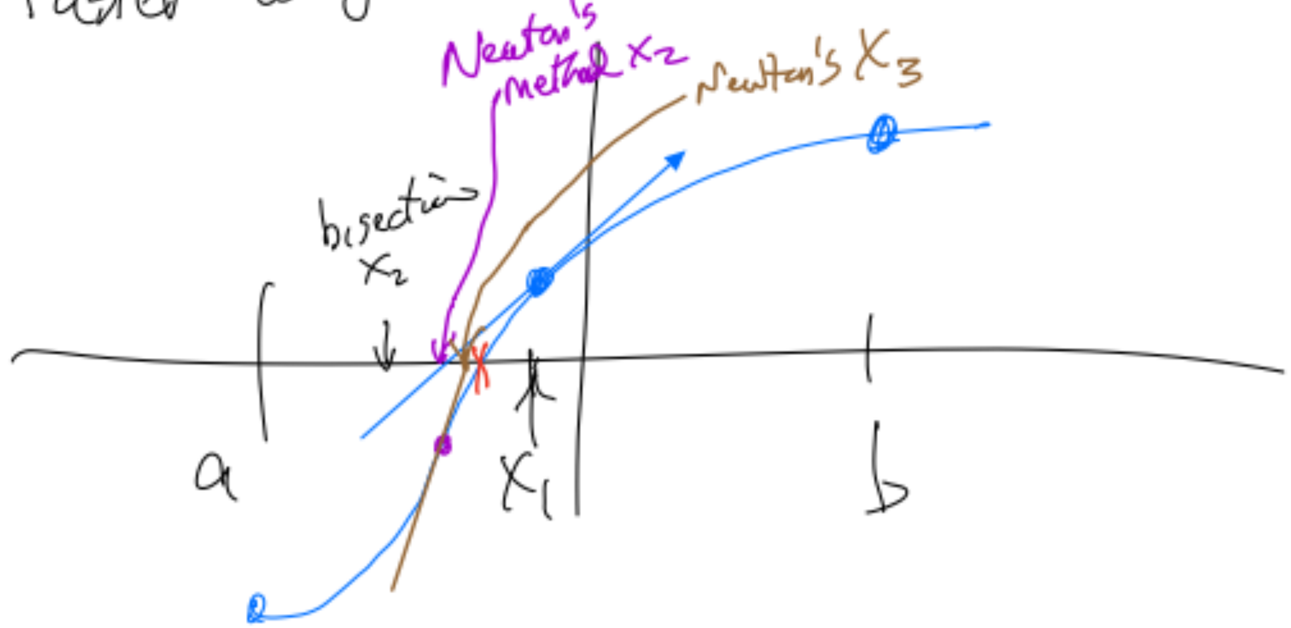
$$f(a) < 0 < f(b),$$

next guess should be

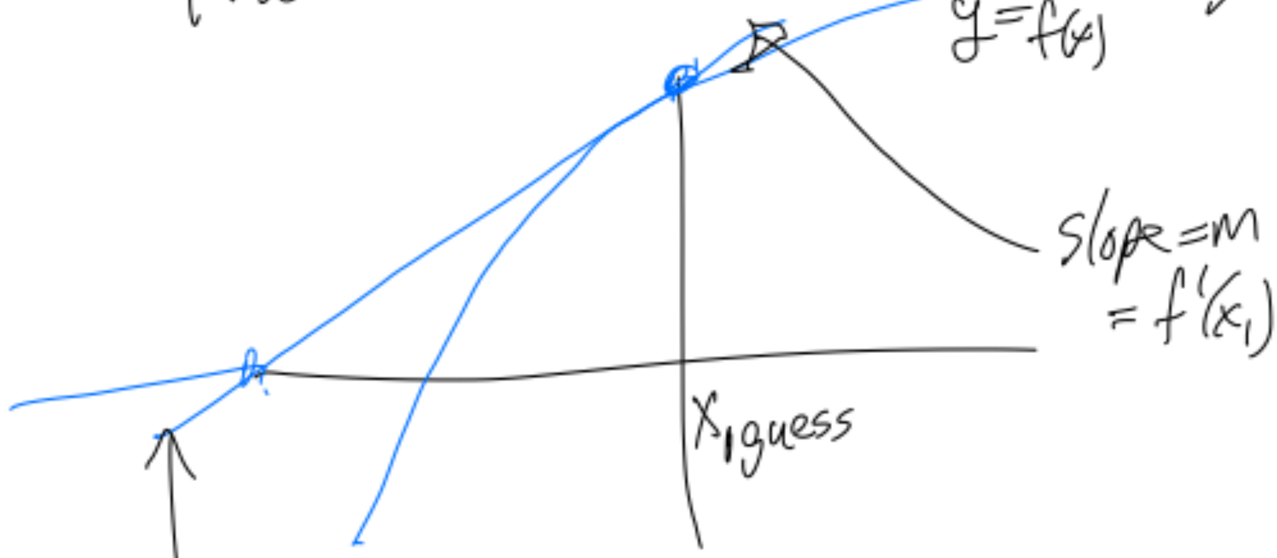
$$x = \frac{a+b}{2}, \text{ plug in.}$$

Then you have an interval half the size — and there is a point between where the function is zero.

Faster way: Newton's Method.



How do we find the next guess



Tangent line equation

$$y - \overset{f(x_1)}{y_1} = f'(x_1)(x - x_1)$$

$$y = f'(x_1)(x - x_1) + f(x_1)$$

Where does it hit the x-axis?

$$y = 0$$

$$0 = f'(x_1)(x - x_1) + f(x_1)$$

$$-f(x_1) = f'(x_1)(x - x_1)$$

$$-\frac{f(x_1)}{f'(x_1)} = x - x_1$$

$$\boxed{x = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

next guess formula for Newton's method.

SageMathCell

Type some Sage code below and press Evaluate.

```
f(x) = 2^x-10*x
fp(x) = diff(f(x),x)
nx(x) = x - f(x)/fp(x)
#initial guess
x0 = 0
x1 = nx(x0)
x2 = nx(x1)
x3 = nx(x2)
x4 = nx(x3)
x5 = nx(x4)
show(x1.n())
show(x2.n())
```

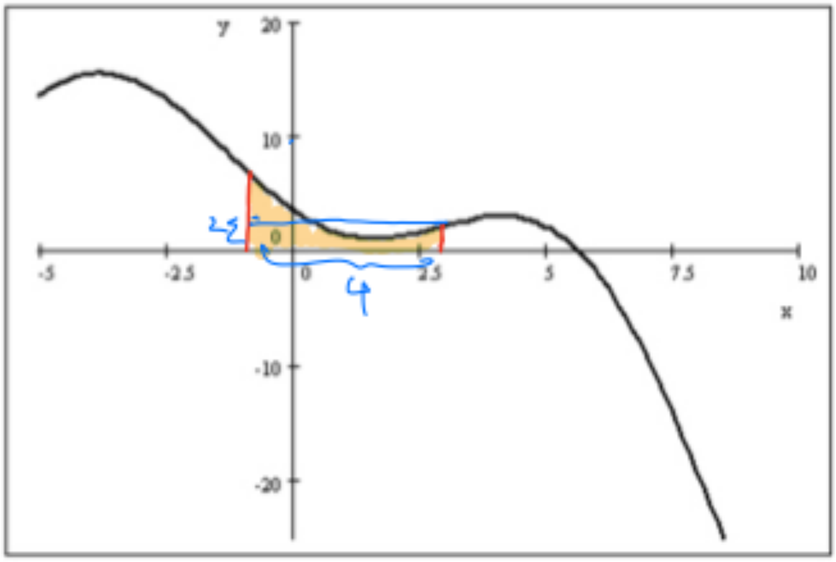
Evaluate

```
0.107447707554933
0.107755012358786
0.107755015000272
0.107755015000272
0.107755015000272
```

fast convergence.

On review for final exam:

(h) $\int_{-1}^{3} (x - 2h(x))\, dx$, where h is the function graphed below.

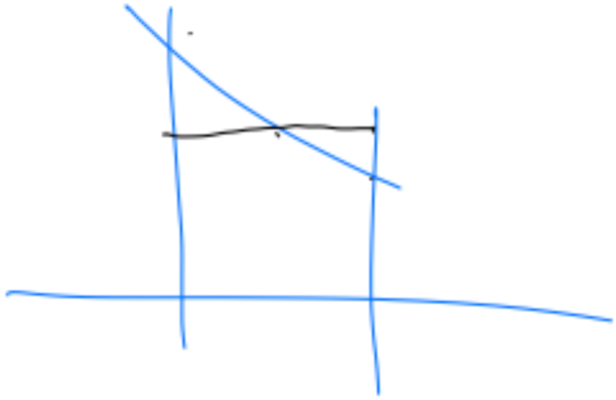


$$\int_{-1}^{3} (x - 2h(x))\, dx = \int_{-1}^{3} x\, dx - 2\int_{-1}^{3} h(x)\, dx$$

$$= \frac{1}{2}x^2 \Big|_{-1}^{3} - 2\,\text{Area}$$

$$= \left(\frac{9}{2} - \frac{(-1)^2}{2}\right) - 2\cdot\text{Area}$$

$$= 4 - 2\cdot\underbrace{\text{Area}}_{8}. = 4 - 16 = \boxed{-12}.$$

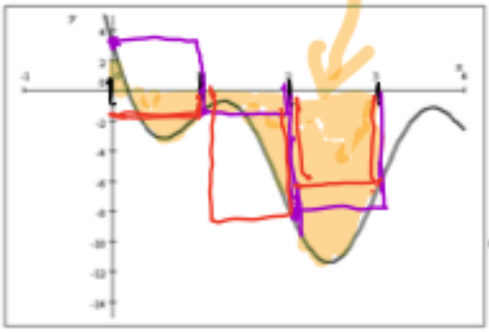


7. Consider the function graphed below.

(a) Estimate the right Riemann sum with three (evenly spaced) subdivisions of the interval $[0, 3]$.

(b) Estimate the left Riemann sum with three (evenly spaced) subdivisions of the interval $[0, 3]$.

(c) Estimate $\int_0^3 h(x)\, dx$ negative



$y = h(x)$

b Left

$(3.5)(1) + (-1.5)(1)$

$+(-8)(1) =$

a Right

$(-1.5)(1) + (-8)(1)$

$+(-6)(1)$